

PRECISION TESTS OF THE MSSM. *

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Abstract

We present the results of a global fit to the electroweak observables in the MSSM in which, for the first time, all the (relevant) low energy parameters of the model are treated as independent variables. The best fit selects either very low or very large values of $\tan\beta$ and chargino (higgsino-like) and stop or/and the CP -odd Higgs boson are within the reach of LEP 2. Moreover, the best fit gives $\alpha_s(M_Z) = 0.114 \pm 0.007$, which is lower than the one obtained from the SM fits. The overall description of the electroweak data is better than in the SM. Those results follow mainly from the fact that in the MSSM one can increase the value of $R_b \equiv \Gamma_{Z^0 \rightarrow \bar{b}b} / \Gamma_{Z^0 \rightarrow \text{hadrons}}$ *without* modifying the SM predictions for other observables.

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Precision tests of the MSSM have been discussed by several groups [1, 2, 3, 4, 5]. In particular, first global fit to the electroweak data within the MSSM parametrized in terms of few parameters at the GUT scale is given in ref. [1]. In ref. [4] the so called ϵ_i parametrization is used and the rôle of light superpartners is studied in some detail. Here we present the results of a global fit to the electroweak observables with, for the first time, all the (relevant) low energy parameters of the MSSM treated as independent variables in the fit [6]. We believe the set of low energy parameters suggested by the precision data may give an interesting hint on physics at the GUT scale.

Our strategy is analogous to the one often used for the SM: in terms of the best measured observables G_F , α_{EM} , M_Z and the less well known m_t , M_h , $\alpha_s(M_Z)$ and a number of additional free parameters in the MSSM such as $\tan\beta$, M_A , soft SUSY breaking scalar masses, trilinear couplings etc. we calculate in the MSSM the observables M_W , all partial widths of Z^0 and all asymmetries at the Z^0 pole. This calculation is performed in the on-shell renormalization scheme [7, 8] and with the same precision as the analogous calculation in the SM, i.e. we include all supersymmetric oblique and process dependent one-loop corrections [9, 27], and also the leading higher order effects.

Similar programme has often been discussed in the context of the SM [10, 11, 12, 2, 13, 23] with the parameters m_t , $\alpha_s(M_Z)$ and M_h (or some of them) to be determined by a fit to the data. Let us first review the results in the SM with the emphasis on those features which are relevant for the supersymmetric extension.

As the experimental input for the observables $1 - M_W^2/M_Z^2$, Γ_Z , σ_h , \mathcal{A}_e , \mathcal{A}_τ , $\sin^2\theta_{eff}^{lept} < Q_{FB} >$, \mathcal{R}_l , $A_{FB}^{0,l}$, R_b , R_c , $A_{FB}^{0,b}$, $A_{FB}^{0,c}$ (i.e. their experimental values, errors and correlation matrices) used in the fits we take the Spring 95 data summarized in ref. [13]. For the top quark mass (which we include in the fit) we use the weighted average of the CDF and D0 result $m_t = (181 \pm 12)$ GeV [14]. For M_W we use the results of ref. [15]: $M_W = 80.33 \pm 0.17$ GeV. (This is the average value of the UA2 measurement and the new measurement reported by the CDF [15]. The D0 collaboration has not published the results of their new analysis yet.)

The Left – Right asymmetry measured by the SLD is $A_{LR} = 0.1551 \pm 0.0040$ [16] and it is included in the fit. For the value of $\Delta\alpha_{EM}^{hadr}$ we use the result of the recent re-analysis in ref. [17]: $\Delta\alpha_{EM}^{hadr} = 0.0280 \pm 0.0007$ with the error propagating in the fit ¹.

In Table 1 we present the results of our global fit in the SM ² (for the

¹The result reported in the ref. [18] has been recently updated [20] and are now closer to the results reported in [17] and those in ref [19] are based on more theoretical assumptions [21].

²Those results agree very well with another recent fit [22].

sake of later discussion in version B we include in the fit the low energy measurement of α_s [26]: $\alpha_s(M_Z) = 0.112 \pm 0.005$.

Table 1. Results of a fit in the SM. All masses in GeV.

fit	m_t	Δm_t	M_h	ΔM_h	$\alpha_s(M_Z)$	$\Delta\alpha_s(M_Z)$	χ^2	d.o.f
A	171.3	$^{+11.5}_{-9.7}$	66	$^{+117}_{-45}$	0.123	0.005	12.6	12
B	172.0	$^{+10.5}_{-9.3}$	59	$^{+96}_{-37}$	0.120	0.005	15.5	13

The fitted value for M_h results in a very transparent way from a combination of effects which can be organized into the following two-step description [23]. A fit to M_W and to all measured electroweak observables but $R_b \equiv \Gamma_{Z^0 \rightarrow \bar{b}b} / \Gamma_{Z^0 \rightarrow \text{hadrons}}$ gives χ^2 values which are almost *independent* of the value of the top quark mass m_t in the broad range (150–200 GeV) and with the best value of $\log M_h$ which is almost linearly correlated with m_t . This is shown in Fig.1. The $m_t - M_h$ correlation is the most solid result of the fits which does not depend on whether R_b and/or m_t measurement of the CDF and D0 [14] are included in the fits. It points toward relatively light Higgs boson for m_t in the range (170 – 190) GeV.

A visible χ^2 dependence as a function of m_t and, therefore, indirect (by constraining m_t) relevant overall limit on the Higgs mass M_h is introduced by the results for R_b and m_t . This is also clearly seen in Fig.1.

Another point of recent interest is the value of $\alpha_s(M_Z)$ obtained from the electroweak fits. It is somewhat larger then the value obtained from low energy data [26]. It is interesting to repeat the SM fit with the low energy measurement $\alpha_s(M_Z) = 0.112 \pm 0.005$ included in the fit.³ Those results are also shown in in Table 1 (case B). The parameters of the fit remain almost unaltered but the overall χ^2 is larger by ~ 3 .

Finally we can interpret the SM fits as the MSSM fits with all superpartners heavy enough to be decoupled. Supersymmetry then just provides a rationale for a light Higgs boson: $M_h \sim \mathcal{O}(100 \text{ GeV})$. and we can expect that the MSSM with heavy enough superpartners gives as good a fit to the precision electroweak data as the SM, with $m_t \sim 170\text{--}180 \text{ GeV}$ (depending slightly on the value of $\tan\beta$).

This is seen in Fig.2 where we show the χ^2 values in the MSSM in both versions, A and B, with the proper dependence of M_h on m_t , $\tan\beta$ and SUSY parameters included [24], with fitted m_t and $\alpha_s(M_Z)$ and with all SUSY mass parameters fixed at 500 GeV. The χ^2 values in the minima are

³One can argue that the determination of $\alpha_s(M_Z)$ based on the deep inelastic (Euclidean) analysis is more precise than from the experiments in the Minkowskean region (jet physics, τ decays). Low value of $\alpha_s(M_Z)$ is also consistent with lattice calculation and has some theoretical support (for review of all those points see M.Shifman, ref.[25]). So, with proper attention to the unsettled controversy and to the fact that jet physics and τ decays give larger values, we are going to explore the assumption that the low energy determination of $\alpha_s(M_Z)$ is the correct one.

very closed to the SM three parameter $(m_t, M_h, \alpha_s(M_Z))$ fit. The only difference is in the m_t dependence of χ^2 : the minimum in the MSSM fit is for slightly larger m_t and simultaneously the upper bound on m_t is more stringent ($m_t < 188$ GeV at 95% C.L.). This is easy to understand as due to the very constrained M_h in the model and to $M_h - m_t$ correlation needed to fit the data ⁴.

Although the SM fit and the MSSM fit with heavy superpartners are globally good, it has been noticed that they cannot properly account for the measured value of R_b which remains almost 3σ higher than the theoretical prediction. Moreover it is well known that new physics in $\Gamma_{Z^0 \rightarrow b\bar{b}}$ and therefore additional contribution to the total hadronic width of the Z^0 boson would lower ⁵ the fitted value of $\alpha_s(M_Z)$ [25, 12], in better agreement with its determination from low energy data [26].

Thus it is conceivable that the measurement of R_b is not a statistical fluctuation but an evidence for new physics and it is very interesting to perform a global fit to the electroweak observables in the MSSM with supersymmetric masses kept as free parameters. In particular we can ask the following two questions [6]:

- a) can we improve R_b without destroying the excellent fit to the other observables?
- b) if we achieve this goal, what are the predictions for sparticle masses?

We begin with a brief overview of the SUSY corrections to the electroweak observables. Although the MSSM contains many free SUSY parameters several of them are irrelevant. Here we list the fitted parameters. These are: m_t , $\alpha_s(M_Z)$, $\tan\beta$, M_A , μ , M_{g2} (we use the relation $M_{g1} = (5/3)\tan^2\theta M_{g2}$; it is of little importance for the results of the fit but fixes the parameters of the lightest supersymmetric particle LSP), $m_{\tilde{q}_L}$ (the soft mass term for the third generation left-handed squarks), $m_{\tilde{b}_R}$, $m_{\tilde{t}_R}$, A_b , A_t and $m_{\tilde{l}_L}$ (a common soft mass parameter for all left handed sleptons). The gluino mass, the first two generation squark masses and the right handed slepton masses are irrelevant for the fit and are always kept heavy. Furthermore, there are remarkable regularities in SUSY corrections [6]. Following our strategy of calculating all electroweak observables in terms of G_F , M_Z and $\alpha_{EM}(0)$ one can establish the following “theorems” for the predictions in the MSSM:

- 1.) $(M_W)^{MSSM} \geq (M_W)^{SM}$. As explained in ref. [27], its origin lies mainly in additional sources of the custodial $SU_V(2)$ violation in the squark and slepton left-handed mass matrix elements (we denote them with capital let-

⁴Scanning over M_A does not change this result as significantly lower values of M_A are excluded by $b \rightarrow s\gamma$ and/or worsen the fit due to negative contribution to R_b .

⁵In the electroweak fits the value of $\alpha_s(M_Z)$ is very precisely determined by strong corrections to the total hadronic Z^0 width. This quantity is calculated with high precision (up to $\mathcal{O}(\alpha_s^3)$) and the experimental error is also very small: $\Gamma_h = 1744.8 \pm 3.0$ [21].

ters e.g. $M_{\tilde{t}_L}^2 = m_{\tilde{q}_L}^2 + m_t^2 + t_\beta(M_Z^2 - 4M_W^2)$ and similarly for the other squarks and sleptons):

$$M_{\tilde{t}_L}^2 - M_{\tilde{\nu}}^2 = t_\beta M_W^2, \quad M_{\tilde{t}_L}^2 - M_{\tilde{b}_L}^2 = m_t^2 - m_b^2 - t_\beta M_W^2 \quad (1)$$

where $t_\beta \equiv (\tan^2 \beta - 1)/(\tan^2 \beta + 1)$, which contribute to $\Delta\rho$ with the same sign as the t - b mass splitting. It should be stressed that the supersymmetric prediction for M_W is merely sensitive to $m_{\tilde{t}_L}$ and $m_{\tilde{q}_L}$ which determine the magnitude of the splitting in eq.(1) relative to the masses $M_{\tilde{t}_L}$ etc. The dependence on the right-handed sfermion masses enters only through the left-right mixing. This also means that the predicted M_W is almost insensitive to the masses of squarks of the first two generations: in their left-handed components there is no source of large $SU_V(2)$ violation. Also, the predictions for M_W are rather weakly dependent on the chargino and neutralino masses m_{C^\pm} , m_{N^0} and the Higgs sector parameters⁶.

2.) Another effect of supersymmetric corrections is that $(\sin^2 \theta^l)^{MSSM} \leq (\sin^2 \theta^l)^{SM}$ where $\sin^2 \theta^l$ can be determined from the on-resonance forward-backward asymmetries

$$A_{FB}^{0l} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_l \quad \text{where} \quad \mathcal{A}_f = \frac{2x_f}{1+x_f^2} \quad (2)$$

with $x_f = 1 - 4|Q_f| \sin^2 \theta^f$. In general, in the on-shell renormalization scheme and with the loop corrections included we get:

$$\sin^2 \theta^l = \left(1 - \frac{M_W^2}{M_Z^2}\right) \kappa_{UN} (1 + \Delta\kappa_{NON}) \quad (3)$$

where κ_{UN} contains universal ‘‘oblique’’ corrections and $\Delta\kappa_{NON}$ – genuine (nonuniversal) vertex corrections which are in this case negligibly small. By using explicit form of κ_{UN} one can derive the following relation:

$$(\sin^2 \theta^l)^{MSSM} = (\sin^2 \theta^l)^{SM} \times \left[1 - \frac{c_W^2}{c_W^2 - s_W^2} (\Delta\rho)^{SUSY} + \dots\right] \quad (4)$$

We see therefore, that the supersymmetric predictions for $\sin^2 \theta^l$ are correlated with the predictions for M_W through the value of $\Delta\rho$ and they are sensitive to the same supersymmetric parameters.

3.) Similarly, the asymmetries in the quark channel are given by the product

$$A_{FB}^{0q} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_q \quad (5)$$

⁶This is due to generically weak $SU_V(2)$ breaking effects in these sectors.

If $(\sin^2 \theta^l)^{MSSM} = (\sin^2 \theta^l)^{SM} - \varepsilon$ and $(\sin^2 \theta^b)^{MSSM} = (\sin^2 \theta^b)^{SM} - \delta$ then it is easy to show that

$$(A_{FB}^{0\ b})^{MSSM} = (A_{FB}^{0\ b})^{SM} \times \left(1 + \frac{\varepsilon}{1 - 4 \sin^2 \theta^l} + 0.2\delta\right) \quad (6)$$

Thus, supersymmetric corrections to $A_{FB}^{0\ b}$ are essentially determined by the corrections to $\sin^2 \theta^l$ and give the third “theorem”: $(A_{FB}^{0\ b})^{MSSM} \geq (A_{FB}^{0\ b})^{SM}$.

At this point it is important to observe that the trends in the MSSM expressed by the above three theorems can only make the comparison of the MSSM predictions with the data worse than in the SM (as for $m_t > 170$ GeV they go against the trend of the data!). Thus we can expect to get lower limits on the left-handed squark and slepton masses, which are the parameters most relevant for the observables M_W , $\sin^2 \theta^l$ and $A_{FB}^{0\ b}$ (of course, for large enough masses, we recover the SM predictions). These limits are amplified by the dependence of Γ_Z on $m_{\tilde{q}_L}$ and $m_{\tilde{l}_L}$. As discussed above, the sensitivity of the each one of those observables to the remaining parameters is weak but may become nonnegligible in the global fit. The most important and interesting is the dependence on the chargino mass and its composition. One can see that a light higgsino (and only higgsino) does not worsen the predictions for M_W , $\sin^2 \theta^l$, $A_{FB}^{0\ b}$ and, for a heavy top quark and light Higgs boson it can significantly improve the fit to Γ_Z [28] due to the Z^0 -wave function renormalization effect which acts similarly to a heavier Higgs boson for heavier t quark (i.e. its contribution makes Γ_Z smaller).⁷

Finally, let us discuss the corrections to the $Z^0 \rightarrow \bar{b}b$ vertex which contribute to the observable R_b .

In the MSSM there are three types of important corrections to the vertex $Z^0 \bar{b}b$ [29]:

- a) charged and neutral Higgs boson exchange [30]; for low $\tan \beta$ and light CP -odd Higgs boson A^0 this contribution is negative (the $\Gamma_{Z \rightarrow \bar{b}b}$ is decreasing below its SM value) whereas for very light A^0 (50 – 80 GeV) and very large values of the $\tan \beta$ (~ 50) the interplay of charged and neutral Higgs bosons is strongly positive;
- b) chargino – stop loops; for heavy top quark and small $\tan \beta$ (i.e. for a given m_t – maximally large top quark Yukawa coupling) they can contribute significantly (and positively) for light chargino (if higgsino-like) and light right-handed top squark (this follows from the Yukawa chargino–stop–bottom coupling); in the case of large $\tan \beta$ this contribution is smaller than for small $\tan \beta$ but the total contribution to the $Z^0 \bar{b}b$ vertex can be amplified by
- c) neutralino – sbottom (if light) loops.

⁷However, this effect can be masked by the additional contribution to the width from $Z^0 \rightarrow N_i^0 N_j^0$ (see later).

Thus, in the MSSM the value of R_b can in principle be significantly larger than in the SM for very low or very large values of $\tan\beta$, light (higgsino-like) chargino, and \tilde{t}_R and/or very light A^0 (for large $\tan\beta$) [31]. It is insensitive to $m_{\tilde{q}_L}$ and $m_{\tilde{l}_L}$.

In summary, in MSSM the electroweak observables exhibit certain “decoupling”: all of them but R_b are sensitive mainly to the left-handed slepton and the third generation squark masses and depend weakly on the right-handed squark masses, gaugino and Higgs sectors; on the contrary, R_b depends strongly just on the latter set of variables and very weakly on the former. We can then indeed expect to increase the value of R_b without destroying the perfect fit of the SM to the other observables. However, chargino, right-handed stop and charged Higgs boson masses also are crucial variables for the decay $b \rightarrow s\gamma$ and this constraint has to be included (there is a sizable uncertainty in the theoretical prediction for $BR(b \rightarrow s\gamma)$ [32, 33] which is taken into account in this paper ⁸).

Of course the obvious constraint for our fits are the present experimental lower bounds for superpartner masses. For chargino and stop we take them to be 47 GeV. In addition, in the parameter space which gives light higgsino-like charginos also neutralinos are higgsino-like and therefore the contribution of $Z^0 \rightarrow N_i^0 N_j^0$ to the total Z^0 width is important in the fit (we also impose the constraint that N_1^0 is the LSP). Another important constraint follows from non-standard top quark decays such as $t \rightarrow \tilde{t} N_i^0$ for a light stop \tilde{t} . The discovery of the top quark in Fermilab through standard decay modes puts upper bound on the BR for non-standard top decays which is of the order of 50% [34].

Let us present some quantitative results. We have fitted the value of α_s , $\tan\beta$, m_t and SUSY parameters listed earlier in two versions A and B (without and with the low energy value $\alpha_s(M_Z) = 0.112 \pm 0.005$ in the fit). The dependence of the χ^2 on $\tan\beta$ for several values of m_t (and scanned over the other parameters) is shown in Fig.3. The best fit is obtained in two regions of very low (close to the quasi-IR fixed point for a given top quark mass, see e.g. [35] and references therein) and very large ($\sim m_t/m_b$) $\tan\beta$ values (for early discussion of large $\tan\beta$ region see [36]). The results are summarized in Table 2.

⁸It is interesting to observe that the uncertainty in the renormalization scale μ in the standard formula $C_7^{eff}(\mu) = \eta_7(\mu)C_7(M_W) + \eta_8(\mu)C_8(M_W) + \eta_2(\mu)C_2(M_W)$ (where $\eta_i(\mu)$ are model independent QCD corrections) can give much larger uncertainty (up to factor two) in the full amplitude in the MSSM compared to the SM. This is due to the fact that, with supersymmetric contributions, the first two terms in C_7^{eff} can be positive whereas in the SM all three are negative; in the latter case an increase by about 80% of $\eta_2(\mu)$ when μ changes from $2m_b$ to $m_b/2$ is partially compensated by decreasing η_7 and η_8 ; with positive C_7 , C_8 and negative C_2 both effects can add up.

Table 2. Results of a fit in the MSSM.

fit	$\tan \beta$	m_t	$\alpha_s(M_Z)$	χ^2	R_b
A	IR	178^{+5}_{-8}	$0.116^{+0.006}_{-0.004}$	10.3	0.218
B	IR	177^{+4}_{-6}	$0.114^{+0.004}_{-0.003}$	10.6	0.218
A	m_t/m_b	172^{+8}_{-7}	0.114 ± 0.005	10.2	0.219
B	m_t/m_b	$174^{+6}_{-7.3}$	0.113 ± 0.004	10.2	0.219

We recall (see Fig. 2) that in the fit with all superpartners heavy the best χ^2 values read: $\chi^2 = 13(16)$, for $\tan \beta = 1.4$ and $\chi^2 = 13.3(16)$, for $\tan \beta = 50$ for fits without (with) low energy value for α_s included. We observe that in version A (B) the best values of χ^2 are by 3(6) lower than in the corresponding fits with all superpartners heavy. Clearly, in version A this improvement is mainly due to higher values of R_b whereas in version B also to the fact that the fitted values of $\alpha_s(M_Z)$ are lower than in the fit with heavy superpartners and much closer to the low energy value $\alpha_s(M_Z) = 0.112 \pm 0.005$ which in version B is included in the fit. It is well known that additional contributions to $\Gamma_{Z \rightarrow \bar{b}b}$ lower the fitted value of $\alpha_s(M_Z)$ [25, 12] and this effect is indeed observed in our fits.

In Fig. 4a we show χ^2 values for versions A of the fit as a function of $\alpha_s(M_Z)$ for different values of $\tan \beta$ and in Fig. 4b the global dependence of χ^2 on $\alpha_s(M_Z)$, with the best fit for $\alpha_s(M_Z) = 0.114 \pm 0.007$. The global dependence of χ^2 on m_t is shown in Fig. 5. We get $m_t = 175^{+7}_{-9}$ GeV and 176^{+4}_{-8} GeV for the A and B versions of the fit respectively.

Increase of $\Gamma_{Z \rightarrow \bar{b}b}$ requires light stop and chargino (for low values of $\tan \beta$) or light CP -odd scalar and/or chargino and stop for large $\tan \beta$ and it is bounded from above by the experimental lower limits on the masses of those particles. A light and dominantly right-handed stop

$$M_{t_1}^2 = M_{t_R}^2 - 2\theta^2 M_{t_L}^2, \quad |\theta| = \left| \frac{A_t m_t}{M_{t_L}^2} \right| \ll 1 \quad (7)$$

is obtained for $M_{t_L} \gg M_{t_R}$ and large L-R mixing term $A_t/M_{t_L} \sim \sqrt{(M_{t_R}^2 - M_{t_1}^2)/2m_t^2}$ (we recall that in our notation capital letters denote the full diagonal entries in the sfermion mass matrix). Our fits give upper bounds on the light stop, chargino and CP -odd Higgs boson masses. In version A, when α_s runs free and is fitted only to the electroweak data, the best fit is better than the corresponding fit with all superpartners heavy by only $\Delta\chi^2 \sim 3$ (but then $\alpha_s(M_Z) = 0.123$). So, we obtain strong upper bounds at 1σ level but no 95% C.L. limits. They are shown in Fig. 6a and 7a for the stop and chargino masses in the low $\tan \beta$ region and for the pseudoscalar and chargino masses in the large $\tan \beta$ region, respectively. Stronger bounds are obtained in version B of the fits, i.e. with the low energy value of $\alpha_s(M_Z)$ included in the fits. They are shown in the same Figures.

The strongest bounds are obtained when $\alpha_s(M_Z)$ is fixed to its best fit value and they are shown in Fig.6b and 7b. The dependence of the strength of the bounds on the way we treat α_s in our fits is quite obvious from the earlier discussion of the depth of the minima in χ^2 . It is interesting to note the structure of the bounds in Figures 7: although the 2σ bounds never constrain the pseudoscalar and the chargino masses simultaneously, one of them remains always light.

Finally in Fig.8 we show the lower 1σ and 2σ limits on left-handed sbottom and left-handed slepton masses for different m_t and $\tan\beta$.

The $BR(b \rightarrow s\gamma)$ has been calculated for each point of the fit with the theoretical uncertainty included according to the ref. [33]. Only the points with $BR(b \rightarrow s\gamma)$ within 2σ of the experimental result have been retained. In Fig. 9 we present the scatter plots which illustrate the role of this constraint in the large $\tan\beta$ region. In Fig. 9a the points with $\Delta\chi^2 < 4$ are plotted in the $(R_b, BR(b \rightarrow s\gamma))$ plane and in Fig. 9b we show χ^2 versus $BR(b \rightarrow s\gamma)$. One can see that the requirement of acceptable $BR(b \rightarrow s\gamma)$ rejects part of the points with best R_b and χ^2 but is consistent with a large number of such points.

In general, one obtains acceptable $BR(b \rightarrow s\gamma)$ due to cancellations between W^\pm , H^\pm and C^\pm and \tilde{t}_R loops. The net impact on the allowed parameters space depends quite strongly on the values of m_t and $\alpha_s(M_Z)$. In the approximation of refs. [32] and in the limit of pure higgsino-like chargino and very heavy second stop and second chargino the amplitude for $b \rightarrow s\gamma$ (before QCD corrections) reads:

$$A_{b \rightarrow s\gamma} = \sum_{i=\gamma,g} (A_W^i + A_{H^+}^i + A_C^i) \quad (8)$$

$$A_W^i + A_{H^+}^i = \frac{3}{2} \frac{m_t^2}{M_W^2} f_i^{(1)} \left(\frac{m_t^2}{M_W^2} \right) + \frac{1}{2} \frac{m_t^2}{M_{H^+}^2} f_i^{(2)} \left(\frac{m_t^2}{M_{H^+}^2} \right) \quad (9)$$

$$A_C^i = -\lambda \frac{m_t^2}{m_{C_1}^2} \left[f_i^{(1)} \left(\frac{M_{\tilde{t}_1}^2}{m_{C_1}^2} \right) + \tan\beta \frac{m_{C_1} A_t}{M_{\tilde{t}_L}^2} f_i^{(3)} \left(\frac{M_{\tilde{t}_1}^2}{m_{C_1}^2} \right) \right] \quad (10)$$

The functions $f_{g,\gamma}^{(k)}$ $k=1,2,3$ are defined in [32] and they all take negative values, the factor $\lambda \approx 1$ for small values of $\tan\beta$ and $\lambda \approx 1/2$ for large values of $\tan\beta$. We see that e.g. for large $\tan\beta$, with small M_A , m_C and $M_{\tilde{t}_1}$ (as needed for the largest R_b) acceptable $BR(b \rightarrow s\gamma)$ requires cancellation between $(A_W^i + A_{H^+}^i)$ and A_C^i (which has to be positive) and correlates those masses and the Left-Right mixing angle $\theta_{\tilde{t}} \approx -m_t A_t / M_{\tilde{t}_L}^2$ (up to the experimental and theoretical uncertainties in the $BR(b \rightarrow s\gamma)$). Although we have to cancel large H^+ contribution, due to the large $\tan\beta$

value the mixing angle which is needed remains small in agreement with the angle θ in eq. (7). The first equation in (7) can be satisfied by a proper adjustment of the parameter $M_{\tilde{t}_R}$. This explains the pattern seen in Fig.9.

In summary, the MSSM fit to the electroweak observables is very good. This is mainly due to higher than in the SM values of R_b , without destroying the agreement in the other observables. Moreover the best fit gives $\alpha_s(M_Z) = 0.114 \pm 0.007$, a value which is lower than the one obtained from the SM fits and in agreement with the low energy data. Low value of $\alpha_s(M_Z)$ is correlated with the presence of the additional contribution to the $\Gamma_{Z \rightarrow b\bar{b}}$ [25, 12]. The best fit selects very particular regions of the parameters space: either very low or very large values of $\tan\beta$ and small higgsino-like chargino and right-handed stop masses for low $\tan\beta$ or/and the CP -odd Higgs boson mass for large $\tan\beta$. For the best value of $\alpha_s(M_Z)$ or with the low energy measurement of $\alpha_s(M_Z)$ included in the fit we obtain strong upper bounds at 95% C.L. on the masses of these particles and predict that they are within the reach of LEP2.

Furthermore, the parameter space selected by the fit is interesting from the theoretical point of view. Low and large values of $\tan\beta$ are theoretically most appealing [35, 36]. The hierarchy $M_{\tilde{t}_L} > M_{\tilde{t}_R}$ which is necessary for a good fit in the low $\tan\beta$ region can be viewed as a natural effect of the top quark Yukawa coupling in the renormalization group running from the GUT scale. For a good fit in the large $\tan\beta$ region this hierarchy is less pronounced, in agreement with $Y_t \approx Y_b$. Finally, the hierarchy $\mu \ll M_{g2}$ (i.e. higgsino-like lightest neutralino and chargino) is inconsistent with the mechanism of radiative electroweak symmetry breaking and universal boundary conditions for the scalar masses at the GUT scale in the minimal supergravity model. However, it is predicted in models with certain pattern of non-universal boundary conditions [37].

Qualitatively similar conclusions for small $\tan\beta$ have been also reached in a recent paper [39]. For large $\tan\beta$, value of $\alpha_s(M_Z)$ similar to ours has been obtained recently also in [40].

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Note added. New electroweak data have been presented at the International Europhysics Conference on High Energy Physics (Brussels, 27 July – 2 August, 1995). The main change are the values of R_b and R_c : $R_b = 0.2219 \pm 0.0017$, $R_c = 0.1543 \pm 0.0074$. Since identification of the c quarks is more difficult than of the b quarks, the experimental groups also present the value of $R_b = 0.2206 \pm 0.0016$ obtained under the assumption that R_c is fixed to its SM value $R_c = 0.172$. The results of the present

paper remain unchanged if we adopt the latter value of R_b and disregard the new value of R_c as unreliable. The MSSM cannot explain any significant departure of R_c from the SM prediction [41].

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FIGURE CAPTIONS

Figure 1. Limits in the (m_t, M_h) plane in the SM. Unclosed lines show the 2σ limits from the fit without the R_b and m_t measurements included. Ellipses show the 1σ and 2σ limits from the fit with the R_b and m_t measurements included.

Figure 2. χ^2 as a function of m_t for two values of $\tan\beta$ in the MSSM with heavy superparticles. Solid (dashed) lines show χ^2 without (with) the low energy measurement of α_s ($\alpha_s(M_Z) = 0.112 \pm 0.005$) included in the fit (versions A and B of the fit respectively). For comparison, the corresponding fits in the Standard Model are shown with dotted (dash-dotted) lines.

Figure 3. Dependence of χ^2 on $\tan\beta$ for different values of m_t . Solid and dashed lines correspond to versions A and B of the fit respectively.

Figure 4. a) Dependence of χ^2 on $\alpha_s(M_Z)$ for different values of $\tan\beta$, b) global dependence of χ^2 on $\alpha_s(M_Z)$. Only version A is shown; in version B the results are very similar.

Figure 5. Global dependence of χ^2 on m_t . Solid and dashed lines correspond to versions A and B of the fit respectively.

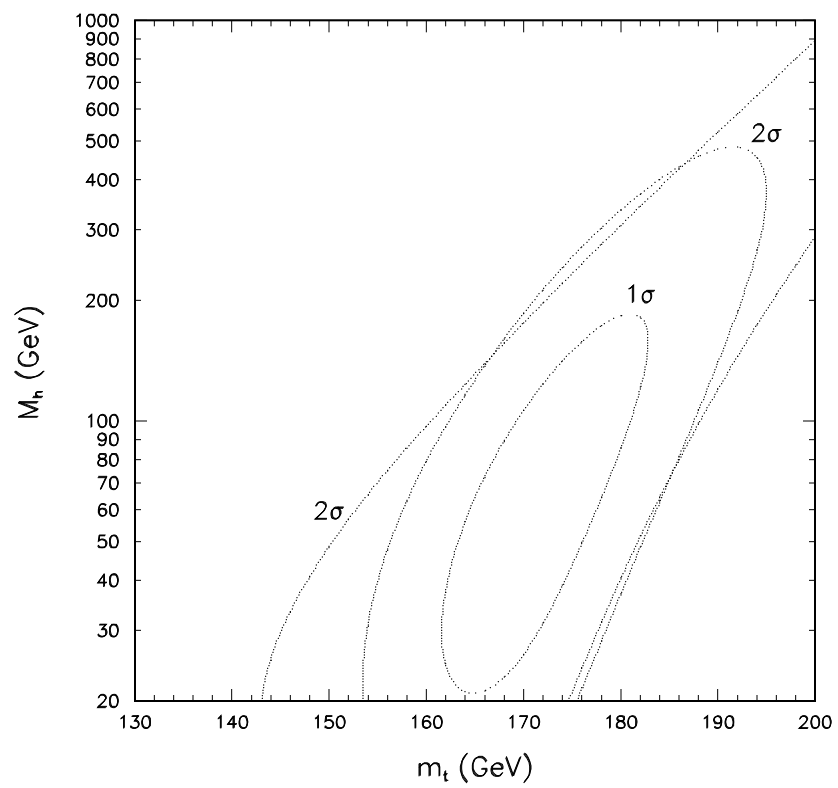
Figure 6. Contours of constant $\Delta\chi^2$ plotted in the chargino – lighter stop mass plane for low $\tan\beta$ fits: a) with $\alpha_s(M_Z)$ fitted as in version B (solid lines) and with $\alpha_s(M_Z)$ free as in version A (dashed lines); b) with $\alpha_s(M_Z)$ fixed to its best value,

Figure 7. Contours of constant $\Delta\chi^2$ plotted in the chargino – CP -odd Higgs boson mass plane for large $\tan\beta$ fits: a) with $\alpha_s(M_Z)$ fitted as in version B (solid lines) and with $\alpha_s(M_Z)$ free as in version A (dashed lines); b) with $\alpha_s(M_Z)$ fixed to its best value.

Figure 8. 1σ and 2σ lower bounds on left-handed sbottom and slepton masses for different values of m_t and $\tan\beta$. Only version A is shown; in version B the results are very similar.

Figure 9. a) Scatter plot in the plane $R_b, BR(b \rightarrow s\gamma)$ of the points with $\Delta\chi^2 < 4$ for $m_t = 170$ GeV and $\tan\beta = 50$; b) χ^2 as a function of $BR(b \rightarrow s\gamma)$ for the same values of m_t and $\tan\beta$.

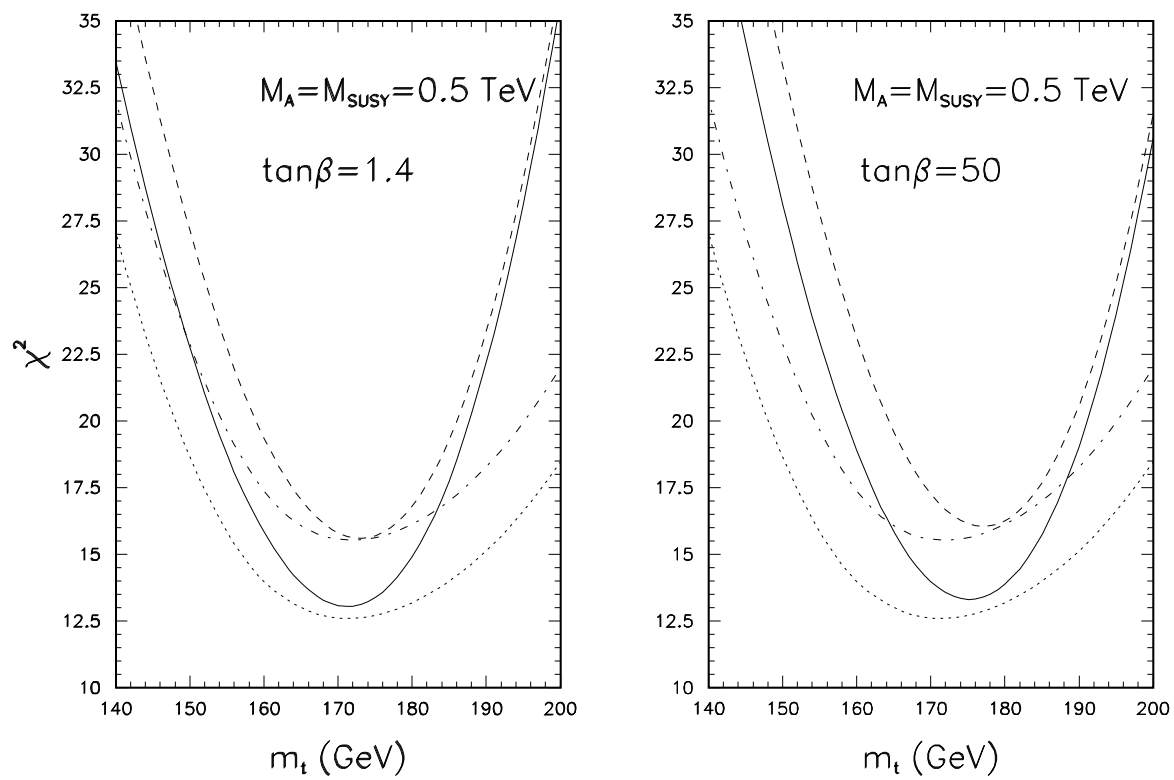
Figure 1.



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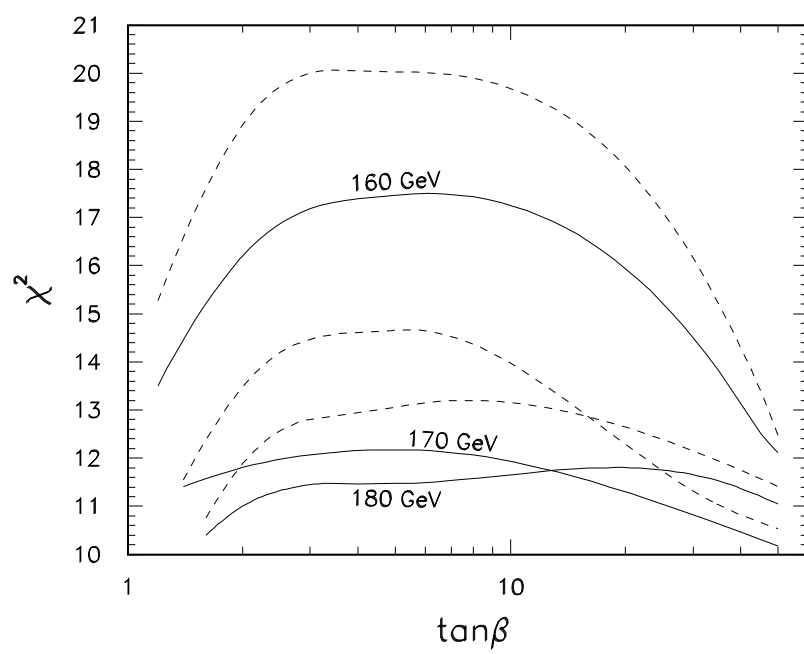
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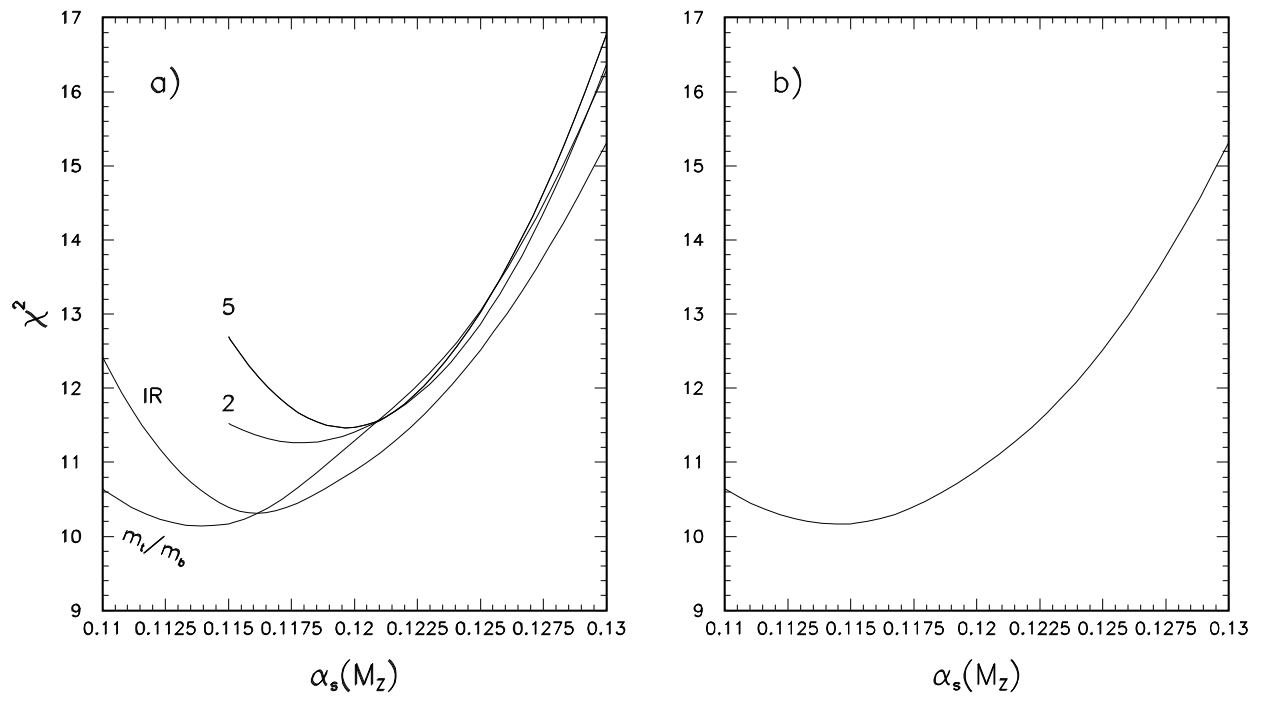
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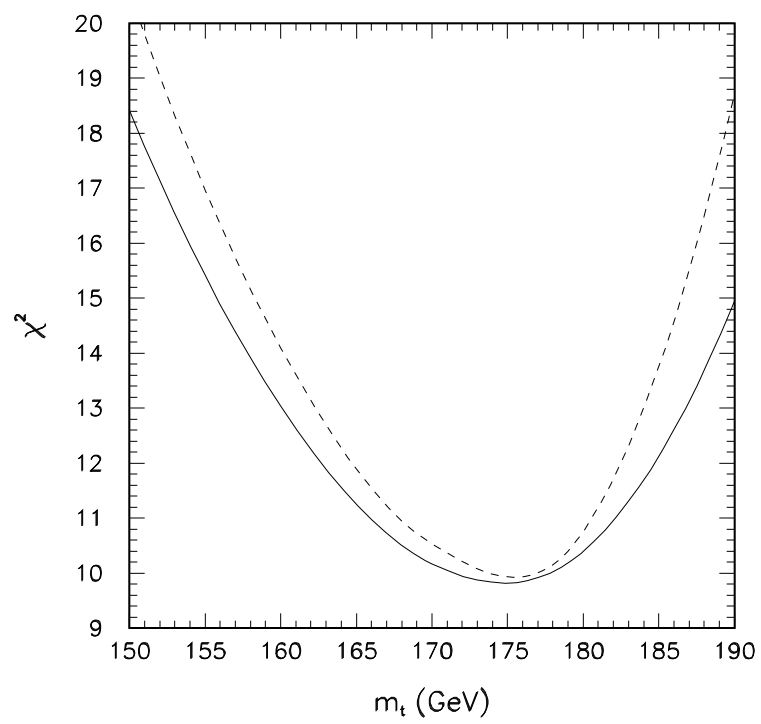
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Figure 5.



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Figure 6

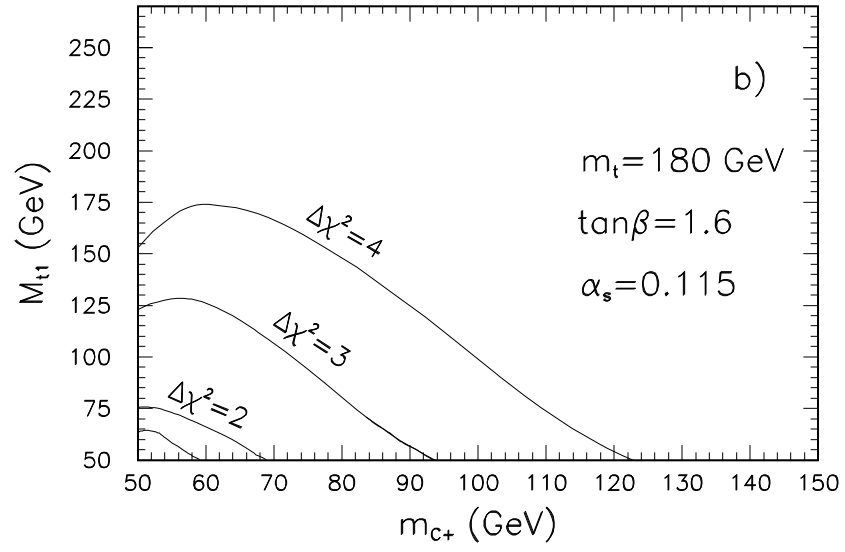
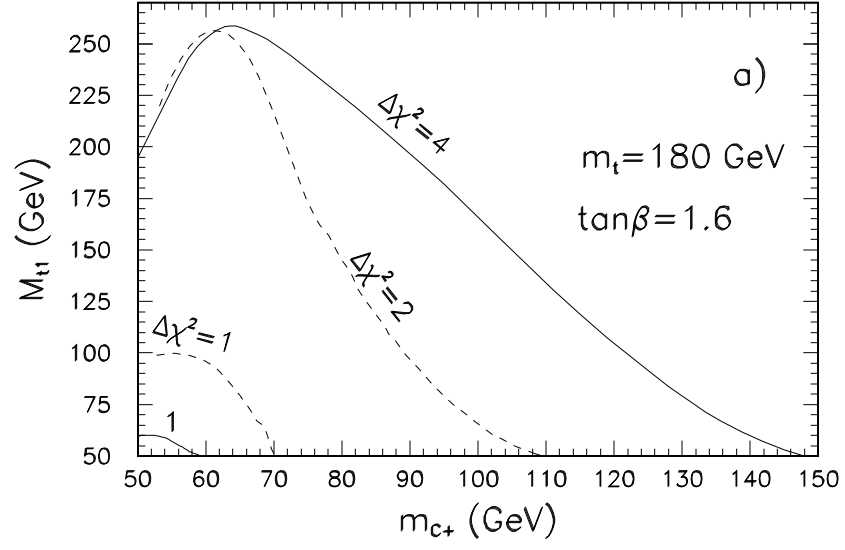


Figure 7

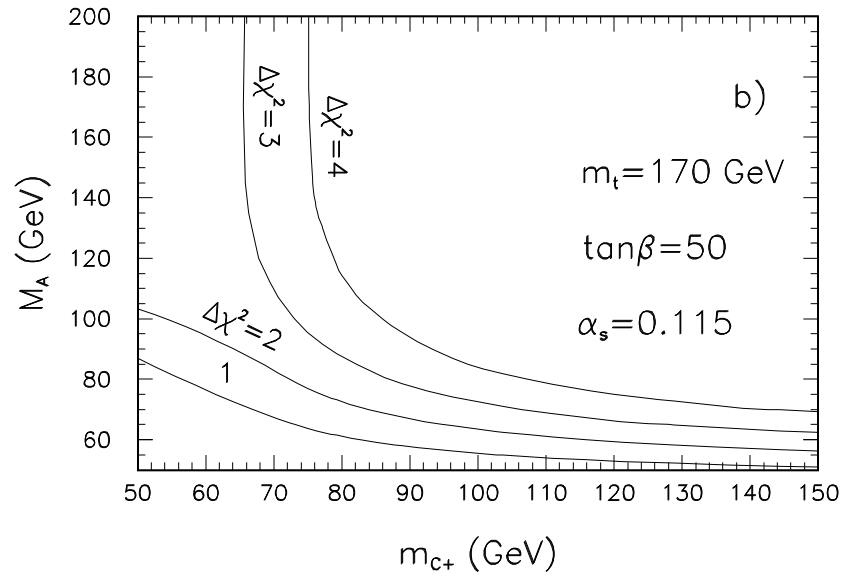
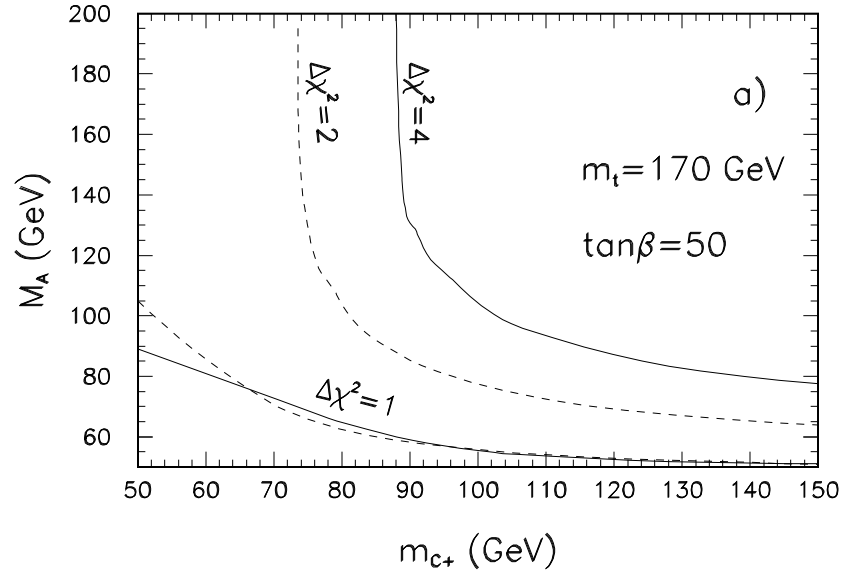


Figure 8.

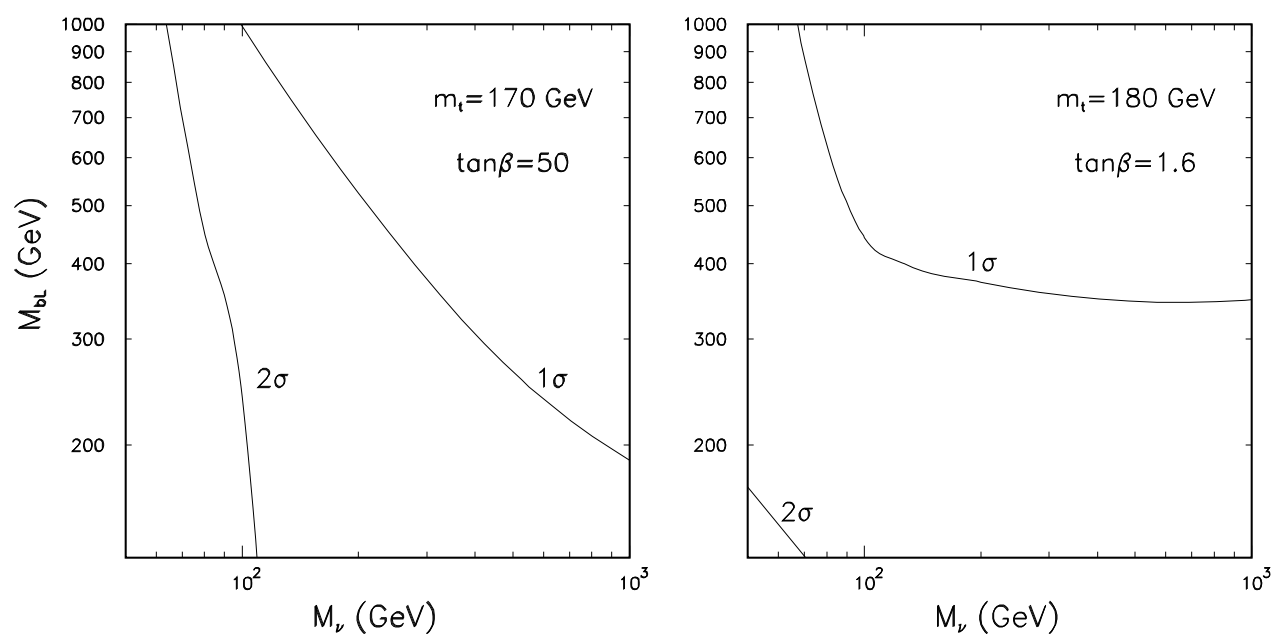


Figure 9

